GEOMETRY AND TOPOLOGY OF TORUS ACTIONS: RESEARCH STATEMENT

The subject of the research is the topological, complex-analytic and symplectic-geometrical study of spaces which arise from well-behaved actions of the torus. This project is a part of the ongoing research in *toric topology*, a new and actively developing field on the borders of equivariant topology, symplectic and algebraic geometry, and combinatorics.

The principal object of the research are *moment-angle manifolds*, and also several related classes of manifolds with torus actions, such as *toric* and *quasitoric* manifolds.

Moment-angle complexes $\mathcal{Z}_K$ are spaces acted on by a torus and parametrised by finite simplicial complexes K. They are central objects in toric topology, and currently are gaining much interest in the homotopy theory. Due the their combinatorial origins, moment-angle complexes also find applications in combinatorial geometry and commutative algebra.

The construction of the moment-angle complex $\mathcal{Z}_K$ ascends to the work of Davis–Januszkiewicz on (quasi)toric manifolds; later $\mathcal{Z}_K$ was described by Buchstaber–Panov as a certain complex build up from polydiscs and tori, and and also as a *coordinate subspace arrangement complement* (up to homotopy). In the case when K is a triangulation of sphere, $\mathcal{Z}_K$ is a topological manifold, referred to as the *moment-angle manifold*. Dual triangulations of simple convex polytopes P provide an important subclass of sphere triangulations; the corresponding *polytopal* moment-angle manifolds are known to be smooth and denoted $\mathcal{Z}_P$. The manifolds $\mathcal{Z}_P$ corresponding to *Delzant polytopes* P are closely related to the construction of *Hamiltonian toric manifolds* via symplectic reduction: $\mathcal{Z}_P$ arises as the level set for an appropriate moment map and therefore embeds into $\mathbb{C}^m$ as a nondegenerate intersection of real quadrics with rational coefficients. The topology of moment-angle complexes and manifolds is quite complicated even for small K and P. The cohomology ring of Z_K was described by Buchstaber and Panov. Later explicit homotopy and diffeomorphism types for certain particular families of $\mathcal{Z}_K$ and $\mathcal{Z}_P$ were described (as wedges of spheres and connected sums of sphere products respectively) in the work of several authors.

On the other hand, manifolds obtained as intersections of quadrics appeared in holomorphic dynamics as the spaces of leaves for holomorphic foliations in $\mathbb{C}^m$. Their study led to a discovery of a new class of compact non-Kähler complex-analytic manifolds in the work of Lopez de Medrano–Verjovsky and Meersseman, now known as the *LVM-manifolds*. Bosio and Meersseman observed that the smooth manifolds underlying a large class of LVM-manifolds are exactly polytopal moment-angle manifolds $\mathcal{Z}_P$. It therefore became clear that $\mathcal{Z}_P$ admits non-Kähler complex-analytic structures generalising the known families of *Hopf* and *Calabi–Eckmann manifolds*.

We plan to concentrate on the following three particular aspects of moment-angle manifolds.

[Topology] Continue the study of topology of moment-angle manifold and complexes. Use homotopy-theoretic methods (higher Whitehead products) to analyse the homotopy and topology types of $\mathcal{Z}_K$ for several important series of polytopes and complexes; in particular those leading to *non-formal* examples of $\mathcal{Z}_P$.

[Complex geometry] Analyse invariants of non-Kähler complex structures on moment-angle manifolds, especially in the non polytopal case. Calculate the ring structure in the Dolbeault cohomology of $\mathcal{Z}_K$ and the Hodge numbers. Derive topological consequences of these calculations.

[Lagrangian geometry] Intersections of real quadrics in $\mathbb{C}^m$ were used by A. Mironov as a starting point in his construction of minimal and Hamiltonian minimal Lagrangian submanifolds in $\mathbb{C}^m$. Since the same intersections of quadrics give rise to moment-angle manifolds, our results on their topology open a way to construct minimal Lagrangian submanifolds with quite complicated topology and understand better the underlying geometry of the Lagrangian embedding.

Moment-angle complexes $\mathcal{Z}_K$ and related toric spaces link together such fields as combinatorial geometry, commutative and homological algebra, and complex geometry. A significant breakthrough in our understanding of the topology and geometry of $\mathcal{Z}_K$ will lead instantly to a host of applications in all these areas.